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ABSTRACT

The purpose of this study was to identify the optimal graduate school admission policy for a medium size state university. The techniques used were Bayesian and cost-benefit decision analysis. Standardized test results, personal interviews with university administrators, and performance of graduate students provide the data necessary for specifying the decision model. The results indicate that graduate admission requirements do not appear to conform to conventional rules of rationality. Moreover, the study indicates that Bayesian/cost-benefit analysis can make a significant contribution to academic policy questions of this type. (Author)

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"AN APPLICATION OF BAYESIAN/COST BENEFIT
ANALYSIS IN ANALYZING GRADUATE SCHOOL
ADMISSIONS POLICY"

by

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1. The first step in the decision-making process is the identification of the problem. This involves a clear definition of the problem and the identification of the relevant factors. The second step is the identification of the alternatives. This involves the generation of a list of possible solutions to the problem.

The third step is the evaluation of the alternatives. This involves the comparison of the alternatives based on the relevant criteria. The fourth step is the selection of the best alternative. This involves the choice of the alternative that is most likely to achieve the desired outcome. The fifth step is the implementation of the selected alternative. This involves the execution of the chosen solution. The sixth step is the evaluation of the results. This involves the assessment of the effectiveness of the chosen solution. The seventh step is the feedback loop. This involves the use of the results to inform future decision-making.

Methodology

The study was conducted in two phases. In the first phase, the researchers identified the problem and the relevant factors. In the second phase, the researchers identified the alternatives and evaluated them based on the relevant criteria.

1. The first step in the decision-making process is the identification of the problem. In our case, the problem was the selection of a graduate student for the University Graduate Council. The relevant factors were the student's academic record, their research interests, and their ability to contribute to the Council.
2. The second step is the identification of the alternatives. All decisions necessarily involved a choice between two or more courses of action. These are referred to as decision acts. The problem is to select the "best" of these alternative decision acts. In this study we assumed that the decision-makers are involved in two distinct decision processes. First, they had to identify the criterion or criteria upon which to base the selection process--i.e., prior performance in academic settings, test scores, or some combination of these criteria. Second, they had to establish cutoff scores for each criterion used.

3. Specific utility or payoff scales have been developed for each individual and for groups of individuals. These scales are based on the assumption that the utility of a particular act is a function of the probability of success in that act. This, of course, is less of a problem in decision-making than in educational decision-making. In business, a large percentage of benefits are realized and measured in dollar terms. This is not true in education. For this reason, educational decision makers are more heavily dependent upon utility scales than their counterparts in the commercial world.

One way to organize the information required in decision analysis is to build a decision matrix. This matrix, expressed symbolically, is given in the following table.

Table I

Alternatives (Acts)	States of Nature (Events)				
	θ_1	θ_2	...	...	θ_m
A_1	u_{11}	u_{12}	...	...	u_{1m}
A_2	u_{21}	u_{22}	...	...	u_{2m}
...	...	...	...	...	...
A_n	u_{n1}	u_{n2}	...	...	u_{nm}

It is assumed that there are n alternative acts, indicated by $A_1, A_2, \dots, A_n$. These acts are listed as row headings in Table I. Also, it is assumed that there are m possible events, indicated by, $\theta_1, \theta_2, \dots, \theta_m$. In the table, the payoffs or utilities are designated by the symbol u with appropriate subscripts. Thus, the symbol u_{ij} denotes the utility of selecting act A_i if event θ_j occurs. This, of course, follows standard rules of matrix notation.

A useful concept in the analysis of decisions is that of opportunity loss or regret. An opportunity loss is the loss incurred if the decision maker acts for other than the best alternative. It is the difference between the payoff or utility of a particular act and the payoff or utility for the optimal act. Under opportunity losses, each of the same solutions to decision problems as the use of profit measures. Because of the convenience in calculation and drafting, opportunity losses rather than utilities were used in this study. To convert Table I to an opportunity loss table the analyst simply identifies the best act A_j for each event B_i and subtracts that value from every value A_{ij} in each column.

Classical cost/benefit analysis differs from Bayesian Decision Analysis in that it assumes that the decision-maker knows before making his decision which state of nature will occur. He assumes the decision is to be made under certainty. This, somewhat artificially, simplifies the decision problem to a choice over the "known" state of nature and selecting the alternative with the highest utility value. Much of classical cost/benefit calculus is concerned with identifying all costs and benefits - reducing them to a common measure of value, and converting to a common time frame. This, of course, is not always an easy task.

The Bayesian decision analyst, on the other hand, follows the classical approach until the selection process. He then explicitly recognizes the uncertainty associated with the occurrence of future events, deals with them through probability assignments and typically applies an actuarial adjustment to the selection process. Each conditional net benefit or utility value is weighted by the likelihood of occurrence of each state of nature, the products are summed across decision acts yielding an expected value for each alternative. The alternative associated with the highest

1. See Schlaifer, Robert Introduction to Statistics for Business Decisions McGraw Hill, 1971 pp 71-79.

expected utility or efficient expected opportunity cost (EOL) is equal to that of the optimal decision.

Section II

In this study, the admission-making group was assumed to have two alternatives with respect to each applicant for graduate school: they could accept the candidate (A_1) or they could reject the candidate (A_2). The "relative goodness" of the choice was assumed to depend upon the likelihood of eventual success (S_1) or lack of success (S_2) in the student's subsequent study program. Success was defined in terms of Graduate GPA earned. A successful student therefore was arbitrarily identified as one whose overall GPA was above the average (mean) GP for all graduate students.

At the time of application to graduate school, the student was required to submit the undergraduate GPA and scores on the graduate admissions examination (verbal, quantitative, and area tests in the Social Sciences, Humanities and Natural Sciences). Simple correlations between each of these measures and graduate GPA are given in Table II.

Table II

Intercorrelations

	1	2	3	4	5
GRE-V 1					
GRE-Q 2	.46*				
GRE-SS 3	.64*	.39*			
GRE-H 4	.56*	.29*	.44*		
GRE-NS 5	.53*	.53*	.49*	.35*	
GPA 6	.32*	.21*	.14	.24*	.15

*Significant at the .01 level.

Table II suggests that overall scores on the combined (quantitative and verbal) Graduate Records test would provide a good indication of

subsequent studies in graduate school. The selection of students for admission to the program is based on a number of factors, including the GRE score, the undergraduate GPA, and the letters of recommendation. The selection process is designed to identify students who are likely to succeed in the program and who will contribute to the academic and professional development of the program.

The selection process is based on a number of factors, including the GRE score, the undergraduate GPA, and the letters of recommendation. The selection process is designed to identify students who are likely to succeed in the program and who will contribute to the academic and professional development of the program. The probability assignments and the performance of the selection process are evaluated in terms of the number of students who are accepted and the number of students who are rejected. The selection process is designed to be fair and to be based on the best available information.

One of the major problems in selection is the problem of the selection process. The selection process is designed to identify students who are likely to succeed in the program and who will contribute to the academic and professional development of the program. The selection process is based on a number of factors, including the GRE score, the undergraduate GPA, and the letters of recommendation. The selection process is designed to be fair and to be based on the best available information.

The problem in establishing the necessary probabilities is, of course, that school records include data only on the students that were accepted into the graduate program. It is relatively easy to establish the marginal probability of acceptance $P(GRE \geq 840)$ or rejection $P(GRE < 840)$. The conditional probabilities of success given acceptance $P(S/GRE \geq 840)$ and the probability of failure given acceptance $P(F/GRE \geq 840)$ can also be determined from available data. It is more difficult but also possible, to estimate

2. Like most traditional programs they also limited enrollment to students with undergraduate GPA of "B" average or above.
3. Hunka, S.J., Cameron D.P. and Pilburt, J.A.L., "Selection of Students for Medical School", The Canadian Medical Association Journal, Vol. 94, pp 708-711, April 1966.

the risk of conditional expected opportunity loss is minimized. Indeed, one can construct a decision rule that will select the strategy that will minimize the conditional expected opportunity loss for each particular strategy (S_j). This rule is called the "Bayes Risk" and is defined as follows:

Using these conditional probabilities and assuming it was decided to establish 340 as a cutoff score in 1954, we can give for the opportunity costs that minimize the risk of this particular strategy.

To have, for example, the "risk" or "conditional expected opportunity loss" is computed by weighting the loss (L_j) by the probability of selection, the cost associated with that loss (L_j) for each particular strategy (S_j). This risk is then weighted by the prior probability of the event yielding the conditional expected opportunity loss. The particular strategy identified with the minimum unconditional expected loss is called "Bayes Risk" and is defined as follows:

The strategies considered were all possible GRE cutoff scores in increments of 10 points. The opportunity loss values that yielded a minimum at 340 are, along with the relevant probabilities, given in Table III below.

Table III

	Conditional Opportunity Losses		Probabilities	
	S ₁ (Success)	S ₂ (Failure)	P(A ₁ /θ ₁)	P(A ₁ /θ ₂)
A ₁ (accept)	0	1(L ₁)	.6758	.7413
A ₂ (reject)	1.55 (L ₂)	0	.1242	.2542

4. Table of Divariance Normal Distribution Function and Related Functions. Washington: National Bureau of Standards, June, 1959.

(1974)

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Domestic Market for Agricultural Products
Options Analysis

This analysis indicates that in 1974 the market for agricultural products was characterized by a number of factors. The first factor was the increase in the price of agricultural products. This was due to a number of reasons, including the increase in the price of oil, the increase in the price of fertilizer, and the increase in the price of labor. The second factor was the increase in the demand for agricultural products. This was due to the increase in the population of the United States and the increase in the demand for food and fiber. The third factor was the increase in the supply of agricultural products. This was due to the increase in the number of farms and the increase in the productivity of the farms.

Since 1974, however, the market for agricultural products has changed. The first change was the decrease in the price of agricultural products. This was due to a number of reasons, including the decrease in the price of oil, the decrease in the price of fertilizer, and the decrease in the price of labor. The second change was the decrease in the demand for agricultural products. This was due to the decrease in the population of the United States and the decrease in the demand for food and fiber. The third change was the decrease in the supply of agricultural products. This was due to the decrease in the number of farms and the decrease in the productivity of the farms.

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Table IV

	Utility Loss		Probability	
	$P(A_1 B_1)$	$P(A_2 B_1)$	$P(A_1 B_2)$	$P(A_2 B_2)$
A_1 (accept)	1	$1/(1+L)$	.5606	.4714
A_2 (reject)	$1/(1+L)$	0	.4393	.5286

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(Conditional expected loss for optimal strategy)

	$P(A_1)$
$B_1 = .2000$	.5 .1044
$B_2 = .7500$	.5 .3672
Unconditional expected loss Optimal Strategy	.4776

Thus, our analysis indicates that while some adjustments in strategy have been made in response to changing societal conditions that most of the adjustment seems to have been offset by a concomitant change in other decision factors. The writers feel that this suggests the present strategy is less than optimal.

Section III

Optimality is an exclusive concept. Its specification depends upon both the utility and probability assessments the decision-maker accepts. Moreover, in a group decision situation it depends upon the decision rules (i.e. unanimous, majority, two-thirds majority, etc.,) adopted by the decision group as well as their individual assessments of the necessary utility and probability relationships. Never the less it is possible to offer some guidelines to optimal strategy for each loss ratio. Table V indicates these relationships letting the ratio of loss values vary from .50 to 2.00 to 1 in increments of .10.

Loss Ratio		Relative Loss		Optimal Cutoff Score
L_1	L_2	L_1/L_2	L_2/L_1	
0.50	.51	.9804	1.0202	1000
1.00	.51	.9950	1.0150	1200
1.00	.75	.9933	1.0167	1300
1.00	.90	.9944	1.0156	1400
1.00	1.00	1.0000	1.0000	1500
1.00	1.10	1.0091	0.9909	1600
1.00	1.20	1.0083	0.9917	1700
1.00	1.30	1.0077	0.9923	1800
1.00	1.40	1.0071	0.9929	1900
1.00	1.50	1.0066	0.9934	2000
1.00	1.60	1.0061	0.9939	2100
1.00	1.70	1.0056	0.9944	2200
1.00	1.80	1.0052	0.9948	2300
1.00	1.90	1.0048	0.9952	2400
1.00	2.00	1.0044	0.9956	2500

It is evident from Table 1 that as the loss ratios change the optimal cutoff score changes as well. We can also see that when the loss associated with the error of accepting a candidate that will fail (L_1) is equal to the loss associated with rejecting a candidate that would succeed (L_2), the optimal cutoff score is 1500 on a scale of 1000 to 2500. The optimal cutoff score increases as the ratio of L_1 to L_2 decreases and decreases as the ratio of L_1 to L_2 increases. One way to establish the specific relative loss values in a decision situation is to attempt to specify them through personal interviews with the decision-makers. Conversations with this group indicates that

